



Running of the scalar spectral index and observational signatures of inflation

James E. Lidsey, Reza Tavakol

Astronomy Unit, School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK

Received 10 April 2003; received in revised form 8 July 2003; accepted 8 July 2003

Editor: J. Frieman

Abstract

Some of the consequences for inflationary cosmology of a scale dependence (running) in the tilt of the scalar perturbation spectrum are considered. In the limit where the running is itself approximately scale-invariant, a relationship is found between the scalar and tensor perturbation amplitudes, the scalar spectral index and its running. This relationship is independent of the functional form of the inflaton potential. More general settings, including that of braneworld cosmological models, are also considered. It is found that for the Randall–Sundrum single braneworld scenario, the corresponding relation between the observables takes precisely the same form as that arising in the standard cosmology. Some implications of the observations failing to satisfy such a relationship are discussed.

© 2003 Elsevier B.V. Open access under [CC BY license](#).

PACS: 98.80.Cq

1. Introduction

Wilkinson microwave anisotropy probe (WMAP) has measured the power spectrum of the cosmic microwave background (CMB) for multipoles up to $l \approx 800$ with unprecedented accuracy [1]. The best-fit model to the WMAP data alone is consistent with a spatially flat universe, with near scale-invariant, adiabatic and Gaussian distributed primordial density (scalar) perturbations [2–4], as predicted by the simplest models of inflation [5]. (For a review, see, e.g., Ref. [6].)

Combinations of CMB results with other astrophysical observations have led to strong constraints on the standard cosmological parameters such as the Hubble parameter, baryon density and age of the universe [2]. In order to differentiate between the numerous inflationary models, however, it is necessary to constrain the power spectrum of the primordial fluctuations. Assuming that such a spectrum varies in a suitable fashion, the standard approach is to expand its logarithm as a Taylor series in $\ln k$, about a given scale, k_0 :

$$\ln A_S(k) = \ln A_S(k_0) + (n_S - 1) \ln \frac{k}{k_0} + \frac{1}{2} \alpha_S \ln^2 \frac{k}{k_0} + \dots, \quad (1)$$

E-mail addresses: j.e.lidsey@qmul.ac.uk (J.E. Lidsey), r.tavakol@qmul.ac.uk (R. Tavakol).

where k is the comoving wavenumber, n_S is the spectral index (tilt) of the spectrum and the second-order term, $\alpha_S \equiv (dn_S/d \ln k)_{k_0}$, represents the ‘running’ of the spectral index [7]. The ‘power-law’ approximation is equivalent to truncating the spectrum to first order, i.e., specifying $\alpha_S = 0$. At this level of approximation, the best-fit to the WMAP data is $n_S = 0.99 \pm 0.04$ [2]. On the other hand, there is some evidence that the power-law approximation may be inadequate when data sets spanning a much wider range of scales are combined. Specifically, Peiris et al. [3] include CMB data from the CBI [8] and ACBAR [9] (covering the range of multipoles $800 < l < 2000$ complementary to WMAP), together with the two degree field (2dF) galaxy redshift survey [10] and Lyman- α forest data at wavenumbers above $k \approx 0.1$ Mpc [11]. There is marginal 1.3σ support for a non-zero running, $\alpha_S = -0.055^{+0.028}_{-0.029}$ [3]. However, the validity of employing the Lyman- α forest data has been questioned [12]. Bridle et al. [13] include CMB data from the VSA [14] but not from the Lyman- α forest, and find the marginalized 1σ result $\alpha_S = -0.04 \pm 0.03$, in agreement with WMAP Collaboration, although they conclude that evidence for a non-zero running is dependent on the surprisingly low values of the quadrupole and octopole moments in the CMB power spectrum. In particular, $\alpha_S = 0$ is consistent when the $l < 5$ multipoles are excluded [13]. Other authors who include only CMB and 2dF data also find that a scale-invariant tilt is consistent with the observations [15–17].

Although there still remain some open questions regarding the interpretation of the data, the recent developments outlined above provide strong motivation for considering what one might expect to learn about inflationary cosmology if a running in the spectral index is detected [7,18–22]. This is especially true given the anticipated improvement in the quality of data from future satellite experiments such as *Planck*. In general inflationary settings one would expect the running itself to be scale-dependent. However, given the current absence of observational evidence for such a variation and, furthermore, that a varying running may be approximated as a piecewise constant over a small enough range of scales, we consider inflationary models where the running of the tilt is scale-independent and non-zero. It is found in Section 2 that the amplitude, tilt and running of the scalar spectrum are related in a non-trivial fashion to the amplitude of the gravi-

tational wave spectrum that is also generated during inflation. A similar relationship is found in Section 3 for a class of braneworld inflationary cosmologies. We conclude with a discussion in Section 4.

2. Running of the spectral index

In general, the power spectrum of the scalar perturbations is closely related to the functional form of the inflaton potential,¹ $V(\phi)$:

$$A_S^2 = \frac{\kappa^6}{75\pi^2} \frac{V^3}{V'^2}, \quad (2)$$

where $\kappa^2 = 8\pi m_P^{-2}$, m_P is the Planck mass and a prime denotes $d/d\phi$. The relationship between the inflaton field and comoving wavenumber follows from the scalar field equations of motion and is given by

$$\frac{d}{d \ln k} = -\frac{V'}{3H^2} \frac{d}{d\phi} \quad (3)$$

in the slow-roll limit. By defining the ‘slow-roll’ parameters as [25]:

$$\epsilon \equiv \frac{1}{2\kappa^2} \frac{V'^2}{V^2}, \quad (4)$$

$$\eta \equiv \frac{V''}{\kappa^2 V}, \quad (5)$$

$$\xi \equiv \frac{V' V'''}{\kappa^4 V^2}, \quad (6)$$

the spectral index and its running may be expressed directly in terms of the potential and its derivatives [7, 26]:

$$n_S - 1 = -6\epsilon + 2\eta, \quad (7)$$

$$\alpha_S = 16\epsilon\eta - 24\epsilon^2 - 2\xi. \quad (8)$$

Thus, the running of the spectral index depends on the third derivative of the potential. Eq. (7) is truncated at order ξ , such that quadratic corrections in ϵ and η are assumed to be negligible. This requires that $|\xi| \ll \max(\epsilon, |\eta|)$ and is equivalent to assuming that $|n_S - 1| \ll 1$ and $|\alpha_S| \approx (n_S - 1)^2$ or less. As emphasized in Ref. [19], slow-roll predicts the former condition but not necessarily the latter.

¹ We employ the normalization conventions of Ref. [24] in this Letter.

The slow-roll parameter (4) is also related to the spectrum of tensor (gravitational wave) perturbations that are generated quantum mechanically during inflation [23]. The relationship is expressed through the consistency equation (for a review, see, e.g., Ref. [24]):

$$\frac{A_T^2}{A_S^2} = -\frac{1}{2}n_T, \quad n_T = -2\epsilon, \quad (9)$$

where $\{A_T, n_T\}$ represent the amplitude and spectral index of the tensor perturbations, respectively.

In view of the discussion given in the previous section, we consider the case where α_S is assumed to be constant. Our aim is to derive an expression relating observable parameters in the presence of a non-zero running. This requires the integration of Eq. (8) with respect to the inflaton field. This equation may be viewed as a third-order, non-linear differential equation. Its first integral therefore relates the inflaton potential to its first two derivatives, or equivalently, the two slow-roll parameters (4) and (5). Consequently, substitution of Eqs. (7) and (9) into such an expression then results in a constraint equation that relates the observable parameters $\{A_S, n_S, \alpha_S, A_T\}$.

To proceed, we introduce a new variable

$$y \equiv \frac{V'}{V} \quad (10)$$

representing the logarithmic derivative of the potential. The third-order equation (8) then reduces to the non-linear, second-order equation

$$yy'' - y^2y' = -\frac{\kappa^4\alpha_S}{2}. \quad (11)$$

Defining $z \equiv y'$, such that $y'' = z dz/dy$, then reduces Eq. (11) to a first-order equation of the form

$$yz \frac{dz}{dy} - y^2z = -\frac{\kappa^4\alpha_S}{2}. \quad (12)$$

Eq. (12) may be further simplified by defining the variable

$$u \equiv z - \frac{1}{2}y^2 \quad (13)$$

and it follows after substitution of Eq. (13) into Eq. (12) that

$$\frac{dy}{du} = -\frac{2}{\kappa^4\alpha_S} \left[uy + \frac{1}{2}y^3 \right], \quad (14)$$

where we now view y and u as the dependent and independent variables, respectively. Eq. (14) may then be rewritten in a separable form by introducing the variable

$$w \equiv y(u) \exp \left[\frac{u^2}{\kappa^4\alpha_S} \right] \quad (15)$$

and substitution of Eq. (15) into Eq. (14) implies that

$$\frac{1}{w^3} \frac{dw}{du} = -\frac{1}{\kappa^4\alpha_S} \exp \left[-\frac{2u^2}{\kappa^4\alpha_S} \right]. \quad (16)$$

Eq. (16) admits the first integral

$$\frac{1}{y^2} \exp \left[-\frac{2u^2}{\kappa^4\alpha_S} \right] - \left(\frac{\pi}{2\kappa^4\alpha_S} \right)^{1/2} \operatorname{erf} \left[\sqrt{\frac{2}{\kappa^4\alpha_S}} u \right] = c, \quad (17)$$

where $\operatorname{erf}(x) \equiv 2\pi^{-1/2} \int_0^x ds \exp[-s^2]$ represents the error function and c is an integration constant. The error function is a monotonically increasing function, such that $\operatorname{erf}(0) = 0$ and $\operatorname{erf}(\infty) = 1$, and has a first derivative given by $d[\operatorname{erf}(x)]/dx = (2/\sqrt{\pi}) \times \exp(-x^2)$. In the case where $\alpha_S < 0$, Eq. (17) may be expressed in terms of the imaginary error function, $\operatorname{erfi}(x) = -i \operatorname{erf}(ix)$.

Finally, the pair $\{u, y\}$ may be directly related to observable parameters. Comparison of Eqs. (4) and (10) implies that $y^2 = 2\kappa^2\epsilon$ and it then follows from definition (13) that the variable u is directly related to the scalar spectral index, $u = \kappa^2(n_S - 1)/2$. Thus, substituting Eq. (9) into Eq. (17) implies that

$$\frac{A_S^2}{A_T^2} \exp \left[-\frac{(n_S - 1)^2}{2\alpha_S} \right] - \left(\frac{2\pi}{\alpha_S} \right)^{1/2} \operatorname{erf} \left[\frac{n_S - 1}{\sqrt{2\alpha_S}} \right] = \tilde{c}, \quad (18)$$

where \tilde{c} is an undetermined constant that is, in principle, measurable. Eq. (18) represents an observable signature of inflationary models that generate a scale-invariant running of the spectral index.

3. Running and braneworld cosmology

It is also of interest to investigate whether the above type of observable signature holds in other more general inflationary scenarios. In recent years, considerable interest has focused on the possibility that our observable four-dimensional universe may be viewed as

a domain wall or ‘brane’ that is embedded in a higher-dimensional ‘bulk’ space [27–29]. According to these scenarios, the standard model gauge interactions are confined to the brane, but gravity may propagate in the bulk [28]. The motion of the brane through the static bulk space is interpreted by an observer confined to the brane as cosmic expansion or contraction [30].

In this section, we consider the Randall–Sundrum type II (RSII) braneworld scenario, where a single brane is embedded in five-dimensional anti-de Sitter (AdS) space [29]. In this case, the effective Friedmann equation on the brane is derived from the Israel junction conditions that relate the extrinsic curvature of the induced metric on the brane to the energy–momentum of the matter fields that are confined to the brane [31]. The form of the Friedmann equation is modified from that of standard cosmology based on Einstein gravity and acquires a quadratic dependence on the energy density, ρ [32]:

$$H^2 = \frac{\kappa^2}{3} \rho \left[1 + \frac{\rho}{2\lambda} \right], \quad (19)$$

where λ is the brane tension.

Such a modification becomes important at high energies and has significant implications for inflation [33,34]. In particular, in the limit where $\rho \gg \lambda$, the amplitudes of the scalar and tensor perturbations are enhanced [33,35]:

$$A_S^2 = \frac{\kappa^6}{600\pi^2} \frac{V^6}{\lambda^3 V'^2}, \quad A_T^2 = \frac{\kappa^4}{200\pi^2} \frac{V^3}{\lambda^2}. \quad (20)$$

However, despite these corrections, the consistency equation relating the two spectra is *identical* to that of the standard scenario, Eq. (9) [36]. Such a degeneracy in the consistency equation also arises in more general brane cosmologies [37].

A natural question to address, therefore, is whether such a degeneracy may be lifted by allowing for a running of the spectral index. In view of the modifications to the Friedmann equation that typically arise in brane cosmology, we consider a model described by a Friedmann equation of the form

$$H^2 = \frac{\tilde{\kappa}^2}{3} \rho^q, \quad (21)$$

where ρ is the energy density of the matter, q is an arbitrary, positive constant and $\tilde{\kappa}^2$ is an arbitrary constant. Eq. (21) may be viewed as a limiting case of

a more generalized Friedmann equation that is relevant in the high energy regime of early universe dynamics. For example, the case $q = 2$ corresponds to the RSII scenario when the energy density dominates the brane tension and we specify $\tilde{\kappa}^2 = \kappa^2/2\lambda$. A further case of interest is given by $q = 2/3$. A Friedmann equation of this form arises in the extended version of the RSII scenario when a Gauss–Bonnet combination of curvature invariants is included in the five-dimensional bulk action [38].

We further assume that the universe is dominated by a single, self-interacting inflaton field. Conservation of energy–momentum of this field then implies that

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \quad (22)$$

where a dot denotes differentiation with respect to time. We define the generalized slow-roll parameters as $\epsilon_g \equiv -\dot{H}/H^2$, $\eta_g \equiv V''/(3H^2)$ and $\xi_g \equiv V'V'''/(3H^2)^2$, respectively, and in the slow-roll limit, $\dot{\phi}^2 \ll V$ and $|\ddot{\phi}| \ll H|\dot{\phi}|$, these reduce to

$$\epsilon_g = \frac{q}{2\tilde{\kappa}^2} \frac{V'^2}{V^{q+1}}, \quad (23)$$

$$\eta_g = \frac{V''}{\tilde{\kappa}^2 V^q}, \quad (24)$$

$$\xi_g = \frac{V'V'''}{\tilde{\kappa}^4 V^{2q}}. \quad (25)$$

Conservation of energy–momentum implies that the curvature perturbation on uniform density hypersurfaces is conserved on super-Hubble radius scales. This follows as a direct consequence of energy–momentum conservation of the inflaton and is independent of the gravitational physics [39]. It can then be shown that the amplitude of the scalar perturbation spectrum is given by $A_S^2 \propto H^4/\dot{\phi}^2$ in the slow-roll limit [39]. The value of the scalar field is related to the comoving wavenumber through Eq. (3). Substituting the field equations into the expression for the amplitude and differentiating with respect to comoving wavenumber then implies that the scalar spectral index is given by

$$n_S - 1 = -6\epsilon_g + 2\eta_g \quad (26)$$

and the running of the tilt is given by

$$\alpha_S = 16\epsilon_g\eta_g - \frac{12(q+1)}{q}\epsilon_g^2 - 2\xi_g. \quad (27)$$

As in the previous section, our aim is to integrate Eq. (27) under the assumption that the running of the spectral index is constant. It proves convenient to define a new scalar field, φ :

$$\frac{d}{d\varphi} \equiv V^{(1-q)/2} \frac{d}{d\phi} \quad (28)$$

and this implies that Eq. (27) takes the form

$$\begin{aligned} \tilde{\kappa}^4 \alpha_S = 4(q+1) \frac{(V^*)^2 V^{**}}{V^3} - 2(2q+1) \frac{(V^*)^4}{V^4} \\ - 2 \frac{V^* V^{***}}{V^2}, \end{aligned} \quad (29)$$

where a star denotes $d/d\varphi$. Defining the new variable $Y \equiv V^*/V$ then simplifies Eq. (29) to

$$Y Y^{**} + (1-2q) Y^2 Y^* = -\frac{\tilde{\kappa}^4 \alpha_S}{2}. \quad (30)$$

Eq. (30) can be integrated in a similar way to that employed in Section 2 and we therefore omit the details. Eq. (30) reduces to the separable equation

$$\frac{1}{W^3} \frac{dW}{dU} = \left(\frac{1-2q}{\tilde{\kappa}^4 \alpha_S} \right) \exp \left[-\frac{2U^2}{\tilde{\kappa}^4 \alpha_S} \right], \quad (31)$$

where

$$U \equiv Y^* + \left(\frac{1-2q}{2} \right) Y^2, \quad (32)$$

$$W(U) \equiv Y \exp \left[\frac{U^2}{\tilde{\kappa}^4 \alpha_S} \right] \quad (33)$$

and solving Eq. (31) then implies that

$$\begin{aligned} \frac{1}{Y^2} \exp \left[-\frac{2U^2}{\tilde{\kappa}^4 \alpha_S} \right] \\ + \sqrt{\frac{\pi}{2\tilde{\kappa}^4 \alpha_S}} (1-2q) \operatorname{erf} \left[\sqrt{\frac{2}{\tilde{\kappa}^4 \alpha_S}} U \right] = C, \end{aligned} \quad (34)$$

where C is an integration constant.

Comparison of Eqs. (26) and (32) implies that $U = \tilde{\kappa}^2 (n_S - 1)/2$. Moreover, substituting Eq. (28) into the definition of the variable Y implies that $Y^2 = (2\tilde{\kappa}^2/q)\epsilon_g$. It follows, therefore, that Eq. (34) may be expressed in the form

$$\begin{aligned} \frac{1}{\epsilon_g} \exp \left[-\frac{(n_S - 1)^2}{2\alpha_S} \right] \\ + \left(\frac{2\pi}{\alpha_S} \right)^{1/2} \left(\frac{1-2q}{q} \right) \operatorname{erf} \left[\frac{n_S - 1}{\sqrt{2\alpha_S}} \right] = \tilde{C}, \end{aligned} \quad (35)$$

where \tilde{C} is a dimensionless constant.

In braneworld inflationary scenarios, the calculation of the tensor perturbation spectrum is more involved than that of the scalar perturbations because the gravitational waves extend into the bulk dimensions [35]. Consequently, one must consider the tensor perturbations for each specific model. For the RSII scenario, where $q = 2$, substituting Eq. (23) into Eq. (20) implies that $A_T^2/A_S^2 = 3\epsilon_g/2$. Remarkably, therefore, we conclude that when Eq. (35) is expressed in terms of the observables $\{A_S, n_S, \alpha_S, A_T\}$, it reduces to *precisely* the same form as the corresponding relationship for the standard inflationary cosmology, Eq. (18).

4. Discussion

The inflationary scenario has received a great deal of observational support from recent CMB satellite observations [2,3]. From the theoretical perspective, an important problem to address is the origin of the inflaton field within a fundamental underlying theory and, more specifically, the nature of the inflaton potential that drove the accelerated expansion of the very early universe. In the case where the running of the spectral index vanishes, it is well known that the form of the potential leading to a constant spectral index is not unique [24,40]. Indeed, the origin of this degeneracy may be understood from a mathematical point of view by expressing the potential in terms of the derivative $V \equiv dW(\phi)/d\phi$ and rewriting Eq. (7) in the form

$$\frac{W'''}{W'} - \frac{3}{2} \left(\frac{W''}{W'} \right)^2 = \frac{\kappa^2}{2} (n_S - 1). \quad (36)$$

This novel way of expressing the constraint on the potential is illuminating because the left-hand side of Eq. (36) is the Schwarzian derivative of the function $W(\phi)$ [41]. This is the unique elementary function of the derivatives that is invariant under the homographic transformation that corresponds to the group of fractional linear transformations:

$$\tilde{W} = \frac{aW + b}{cW + d}, \quad (37)$$

where $\{a, b, c, d\}$ are arbitrary constants satisfying $ad - bc = 1$. Thus, given a particular solution to Eq. (36) (such as an exponential potential), more

general solutions and corresponding potentials may be generated by applying the transformation (37). Further observational input, most notably from the gravitational wave background, is required to lift the degeneracy [24].

In this Letter we have considered the more general class of inflationary models where the running of the scalar spectral index is itself non-zero and independent of scale. In general, it is not possible to determine the analytic form of the potentials that generate such a spectrum. On the other hand, their asymptotic limit may be deduced by noting that the first integral of the second-order equation (11) may be written in the form

$$y' = \sqrt{f(\phi) - \kappa^4 \alpha_S \ln y}, \quad (38)$$

where the function $f(\phi)$ itself satisfies the non-linear equation, $f' = 2yy'^2$, i.e.,

$$f' = 2y(f - \kappa^4 \alpha_S \ln y). \quad (39)$$

The pair of Eqs. (38) and (39) represent a plane autonomous system with a single equilibrium point located at $f = \kappa^4 \alpha_S \ln y$ and it follows from the definition of the function f that this point represents the asymptotic form of the general solution to Eq. (11) in the limit where $|y''| \ll y|y'|$, i.e., where the first term on the left-hand side of Eq. (11) is negligible. In this limit, Eq. (11) may be integrated to yield the form of the potential:

$$V = V_0 \exp \left[\pm \left(\frac{81|\alpha_S|}{128} \right)^{1/3} (\kappa\phi)^{4/3} \right], \quad (40)$$

where V_0 is an arbitrary positive constant and the sign of the exponent corresponds to the sign of the running. It is worth remarking that potentials of this specific asymptotic form also arise within the context of supergravity models [42].

Eq. (18) directly relates the four observable parameters $\{A_S, n_S, \alpha_S, A_T\}$. An important feature of this relation is that it is *independent* of the specific functional form of the inflaton potential. In this sense, therefore, it represents an observable signature of the class of inflationary models where the ‘running of the running’, $\beta_S \equiv d\alpha_S/d \ln k$, is negligible. However, the expressions for the tilt and running, Eqs. (7) and (8), were derived under the standard assumption that $|\xi| \ll \max(\epsilon, |\eta|)$. Although this is consistent, it is not

required by the slow-roll approximation [19] and failure to satisfy Eq. (18) may therefore indicate that such an assumption would need to be relaxed.

The occurrence of the arbitrary constant implies that to proceed observationally the parameters $\{A_S, n_S, \alpha_S, A_T\}$ must be measured over at least two separate scales. One measurement is required to determine the numerical value of the integration constant and the second to determine whether Eq. (18) is indeed satisfied. The advantage of Eq. (18) over the consistency equation (9) is that it relates the scalar and tensor perturbation amplitudes directly to the scalar spectral index and its running. Consequently, it does not require the tilt of the tensor spectrum to be directly measured.

On the other hand, we have assumed that the running is effectively constant and this can only be verified observationally to within some error. When discussing observational constraints, the primordial power spectrum may be viewed as an unknown function and the fitting procedure effectively truncates the Taylor expansion (1) at a finite order. This is equivalent to setting all corresponding higher-order, slow-roll parameters to zero. Establishing whether a constant running is a good fit to the data requires the introduction of the next-order term, $\beta_S \equiv d\alpha_S/d \ln k$, as an additional parameter in the analysis. Self-consistency of the assumptions made above requires that $\beta_S \approx |n_S - 1|^3$ or smaller. The assumption that the running is constant would then be supported if it turned out that β_S has only a moderate influence on the likelihood distributions for the other observable parameters and is itself entirely consistent with zero within the observed errors.

Nevertheless, Eq. (18) may prove important even if a high running of the running is reported. Leach and Liddle have argued that appropriate conditions should be satisfied if the inclusion of a higher-order parameter of the power spectrum is to be justified [17]. In effect, the criterion is that of convergence in the Taylor expansion (1). In the present context, the inclusion of the running could only be justified if the third-order term in Eq. (1) is significantly smaller than the second-order term and, quantitatively, this requires

$$\left| \frac{\beta_S}{3} \ln \left(\frac{k}{k_0} \right) \right| \ll |\alpha_S|. \quad (41)$$

If condition (41) is violated when the detection of β_S has only a low significance, it could be argued that

the determination of this parameter may be unreliable and that it is therefore not appropriate to include it in the analysis [17]. Indeed, Eq. (18) proves important in this case because it may be employed to yield crucial information about the magnitude of the second derivative of the tilt. Failure of Eq. (18) to satisfy the data (under the assumption of a constant running) could be interpreted as evidence that the second derivative of the tilt is important without the need for allowing this parameter to be a priori non-zero.

We have also considered inflationary models that generate a scale-invariant running of the spectral index in the Randall–Sundrum type II braneworld scenario. Surprisingly, we found that in the high energy limit, the relation (35), when expressed in terms of the observables $\{A_S, n_S, \alpha_S, A_T\}$, takes *precisely* the same form as the corresponding relationship for the standard inflationary cosmology, Eq. (18). This provides further evidence of the degeneracy that exists between the primordial perturbations that are generated in the two scenarios even though the gravitational physics is manifestly different in the two cases [36,37]. It should be borne in mind, however, that in these calculations the effects of the bulk space on the evolution of the density perturbations has been neglected. This is consistent at linear order when considering scalar perturbations of a homogeneous background [33]. More generally, the backreaction perturbs the bulk space away from conformal invariance and generates a non-trivial Weyl curvature in the bulk [43,44]. This plays the role of a non-local energy–momentum source when projected down to four dimensions and thus alters the background dynamics [43–45]. The failure of the relation (18) to be satisfied in this case could therefore indicate the possible importance of these bulk effects.

Acknowledgements

J.E.L. is supported by the Royal Society. We thank N. Nunes for helpful discussions.

References

- [1] C. Bennett, et al., astro-ph/0302207.
- [2] D.N. Spergel, et al., astro-ph/0302209.

- [3] H.V. Peiris, et al., astro-ph/0302225.
- [4] E. Komatsu, et al., astro-ph/0302223.
- [5] V. Mukhanov, G. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. 33 (1981) 549, JETP Lett. 33 (1981) 532; A.H. Guth, S.Y. Pi, Phys. Rev. Lett. 49 (1982) 1110; S. Hawking, Phys. Lett. B 115 (1982) 295; A.A. Starobinsky, Phys. Lett. B 117 (1982) 175; A.D. Linde, Phys. Lett. B 116 (1982) 335; J.M. Bardeen, P.J. Steinhardt, M.S. Turner, Phys. Rev. D 28 (1983) 679.
- [6] A.R. Liddle, D.H. Lyth, Cosmological Inflation and Large-Scale Structure, Cambridge Univ. Press, Cambridge, 2000.
- [7] A. Kosowsky, M.S. Turner, Phys. Rev. D 52 (1995) 1739.
- [8] T.J. Pearson, et al., astro-ph/0205388.
- [9] C.L. Kuo, et al., astro-ph/0212289.
- [10] W.J. Percival, et al., Mon. Not. R. Astron. Soc. 327 (2001) 1297.
- [11] R.A.C. Croft, et al., Astrophys. J. 581 (2002) 20; N.Y. Gnedin, A.J.S. Hamilton, Mon. Not. R. Astron. Soc. 334 (2002) 107.
- [12] U. Seljak, P. McDonald, A. Makarov, astro-ph/0302571.
- [13] S.L. Bridle, A.M. Lewis, J. Weller, G. Efstathiou, astro-ph/0302306.
- [14] K. Grainge, et al., astro-ph/0212495.
- [15] V. Barger, H. Lee, D. Marfatia, hep-ph/0302150.
- [16] W.H. Kinney, E.W. Kolb, A. Melchiorri, A. Riotto, hep-ph/0305130.
- [17] S.M. Leach, A.R. Liddle, astro-ph/0306305.
- [18] G. Jungman, M. Kamionkowski, A. Kosowsky, D.N. Spergel, Phys. Rev. D 54 (1996) 1332; D.H. Lyth, hep-ph/9609431; E.J. Copeland, I.J. Grivell, A.R. Liddle, Mon. Not. R. Astron. Soc. 298 (1998) 1233; S.M. Leach, A.R. Liddle, J. Martin, D.J. Schwarz, Phys. Rev. D 66 (2002) 023515; B. Gold, A. Albrecht, astro-ph/0301050.
- [19] S. Dodelson, E. Stewart, Phys. Rev. D 65 (2002) 101301; E.D. Stewart, Phys. Rev. D 65 (2002) 103508.
- [20] E.D. Stewart, Phys. Lett. B 391 (1997) 34; E.D. Stewart, Phys. Rev. D 56 (1997) 2019; X. Wang, B. Feng, M. Li, astro-ph/0209242; B. Feng, M. Li, R. Zhang, X. Zhang, astro-ph/0302479.
- [21] J. Gong, E.D. Stewart, Phys. Lett. B 538 (2002) 213; E.W. Mielke, H.H. Peralta, Phys. Rev. D 66 (2002) 123505.
- [22] M.B. Hoffman, M.S. Turner, Phys. Rev. D 64 (2001) 023506; S. Hansen, M. Kunz, Mon. Not. R. Astron. Soc. 336 (2002) 1007; C. Caprini, S.H. Hansen, M. Kunz, Mon. Not. R. Astron. Soc. 339 (2003) 212.
- [23] A.A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. 30 (1979) 719, JETP Lett. 30 (1979) 682; L.F. Abbott, M.B. Wise, Nucl. Phys. B 244 (1984) 541.
- [24] J.E. Lidsey, A.R. Liddle, E.W. Kolb, E.J. Copeland, T. Barreiro, M. Abney, Rev. Mod. Phys. 69 (1997) 373.
- [25] A.R. Liddle, P. Parsons, J.D. Barrow, Phys. Rev. D 50 (1994) 7222.
- [26] A.R. Liddle, D.H. Lyth, Phys. Lett. B 291 (1992) 391.

- [27] K. Akama, in: *Lecture Notes in Physics*, Vol. 176, Springer, Berlin, 1982, p. 267 hep-th/0001113;
V.A. Rubakov, M.E. Shaposhnikov, *Phys. Lett. B* 125 (1983) 136;
M. Visser, *Phys. Lett. B* 159 (1985) 22;
N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* 429 (1998) 263;
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* 436 (1998) 257;
L. Randall, R. Sundrum, *Phys. Rev. Lett.* 83 (1999) 3370;
A. Lukas, B.A. Ovrut, K.S. Stelle, D. Waldram, *Phys. Rev. D* 59 (1999) 086001;
A. Lukas, B.A. Ovrut, D. Waldram, *Phys. Rev. D* 60 (1999) 086001;
M. Gogberashvili, *Europhys. Lett.* 49 (2000) 396.
- [28] P. Hořava, E. Witten, *Nucl. Phys. B* 460 (1996) 506;
P. Hořava, E. Witten, *Nucl. Phys. B* 475 (1996) 94.
- [29] L. Randall, R. Sundrum, *Phys. Rev. Lett.* 83 (1999) 4690.
- [30] P. Kraus, *JHEP* 9912 (1999) 011;
D. Ida, *JHEP* 0009 (2000) 014.
- [31] W. Israel, *Nuovo Cimento B* 44 (1966) 1.
- [32] P. Binétruy, C. Deffayet, D. Langlois, *Nucl. Phys. B* 565 (2000) 269;
P. Binétruy, C. Deffayet, U. Ellwanger, D. Langlois, *Phys. Lett. B* 477 (2000) 285;
E.E. Flanagan, S.-H. Tye, I. Wasserman, *Phys. Rev. D* 62 (2000) 044039;
J.M. Cline, C. Grojean, G. Servant, *Phys. Rev. Lett.* 83 (1999) 4245;
- C. Csáki, M. Graesser, C. Kolda, J. Terning, *Phys. Lett. B* 462 (1999) 34.
- [33] R. Maartens, D. Wands, B. Bassett, I. Heard, *Phys. Rev. D* 62 (2000) 041301.
- [34] E.J. Copeland, A.R. Liddle, J.E. Lidsey, *Phys. Rev. D* 64 (2001) 023509.
- [35] D. Langlois, R. Maartens, D. Wands, *Phys. Lett. B* 489 (2000) 259.
- [36] G. Huey, J.E. Lidsey, *Phys. Lett. B* 514 (2001) 217.
- [37] G. Huey, J.E. Lidsey, *Phys. Rev. D* 66 (2002) 043514;
G.F. Giudice, E.W. Kolb, J. Lesgourgues, A. Riotto, *Phys. Rev. D* 66 (2002) 083512.
- [38] C. Charmousis, J.F. Dufaux, *Class. Quantum Grav.* 19 (2002) 4671;
S.C. Davis, hep-th/0208205;
E. Gravanis, S. Willison, hep-th/0209076.
- [39] D. Wands, K.A. Malik, D.H. Lyth, A.R. Liddle, *Phys. Rev. D* 62 (2000) 043527.
- [40] J.E. Lidsey, R. Tavakol, *Phys. Lett. B* 309 (1993) 23.
- [41] E. Hille, *Ordinary Differential Equations in the Complex Domain*, Wiley, New York, 1976.
- [42] E.J. Copeland, N.J. Nunes, F. Rosati, *Phys. Rev. D* 62 (2000) 123503.
- [43] T. Shiromizu, K. Maeda, M. Sasaki, *Phys. Rev. D* 62 (2000) 024012.
- [44] R. Maartens, *Phys. Rev. D* 62 (2000) 084023.
- [45] C. Gordon, R. Maartens, *Phys. Rev. D* 63 (2001) 044022;
R. Maartens, gr-qc/0101059.